An agent-based model of horizontal mergers

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Abstract. This paper is about the agentization of a horizontal mergers model. In this model, firms are either in a differentiated products Bertrand competition, in which they choose prices in order to maximize their profits, or in a Cournot competition, in which quantities are chosen by firms. The analytical game theoretical model predicts that once a firm merges to another, prices of the merging party rise, which leads to a decrease in consumer surplus and an increase in producer surplus. Developed along this draft is an agent-based version of this model in which firms do not know the demand they are facing. We find convergence of our agent-based model to the game theoretical results before and after firms merge. Alternative learning methods will be implemented as a further extension to this agent-based model.

Keywords: Horizontal Mergers, Agent-based models, Game Theory

1 Introduction

A horizontal merger occurs when competitors in an industry merge. Since this event leads to the concentration of an industry, one of its possible consequences is the exercise of unilateral market power – which could translate into a raise in prices not only of the goods involved in merging parties, but of the whole market. According to the Federal Trade Commission, the regulation agency responsible for evaluating mergers in the United States, over a thousand merger cases are reviewed every year [1]. In Europe, the European Commission (EC) received around 230 notifications until july of this year[2]. As stated by the Federal Trade Commission, around 5% of the cases reviewed by the agency present competitive issues. In that case, to prevent mergers that are damaging to the consumer's welfare and competition, it is necessary to study and develop methods that help verifying if mergers will affect competition negatively.

A significant amount of methodologies that measure the effects of mergers have been developed such as [5], [1], [3] and [2]. All of these methods use analytical and statistical tools for the estimation of prices before and after mergers. However, there's an absence of models that describe and reproduce such effects of unilateral market power in the agent-based computational economics literature. Although industrial organization studies the strategic interaction between firms, which could be represented through agent-based modeling tools, [10] states a lack of integration between the industrial organization theory and agent-based methodologies. This paper is a small contribution to narrow the gap between these two research areas.

[1] See: https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/mergers

[2] See:https://ec.europa.eu/competition-policy/mergers/statistics_en

Using a constructive approach, as proposed by [13], an agent-based model of horizontal mergers is developed. A model presented in [9], where firms are in a differentiated products Bertrand game, is used as a benchmark case and goes through the process of *agentization*, which is also described in [**?**]. The model is also extended to the Cournot game. Assumptions of perfect rationality and information are relaxed from the model, and the emerging patterns before and after mergers are studied. Under our chosen assumptions of agent learning, qualitative and quantitative results are identical to the analytical model.

The work is divided into four sections. In the first section, the exposition and assumptions of the analytical model are presented. The second section presents the agent-based version of this model. Presented in the third section are the results obtained from simulations. Finally, the fourth has concluding remarks about this work and extensions that we plan to include on a future version.

2 The analytical model

As mentioned earlier, this model is largely based on the model of horizontal mergers featured in [9, p.244-265]. To the reader that might not be familiar with the concept of a horizontal merger, these mergers happen whenever firms of the same industry merge[3]. A quick exposition of the model is going to be done in this section, showing the utility of a representative consumer and how it determines quantities given the prices of the goods in the market and how firms, considering this demand, determine optimal prices.

2.1 Bertrand competition with differentiated products

The utility of the representative consumer depends on multiple differentiated products in the market. It is represented as:

$$U = v\sum_{i=1}^{n} q_i - \frac{n}{2+\gamma}\Big[\sum_{i=1}^{n} q_i^2 + \frac{\gamma}{n}\Big(\sum_{i=1}^{n} q_i\Big)^2\Big] + y \tag{1}$$

where y is an outside good, and since this demand is quasi-linear, it does not affect the decisions taken by the consumer with respect to the differentiated products; q_i is the quantity of the i-th product; v is a positive parameter; n is the number of products in the industry; and γ represents the degree of substitutability between the n products. From utility maximization, prices and quantities can be determined. The direct demand function is defined as:

$$q_i = \frac{1}{n}\Big[v - p_i(1+\gamma) + \frac{\gamma}{n}\sum_{j=1}^{n} p_j\Big] \tag{2}$$

[3] One alternative would be the occasion in which a firm sells input to another firm that produces a final good. In the case that one firm merges with the other, this event would be called a vertical merger.

Quick inspection shows that the demand for the i-th good depends on its own price and the price of other goods, that is, if a good has a price that is too high, it will be substituted by other products, depending on the degree of substitutability, which will lead to a diminished demand for this product. However, if other goods are too expensive, then the good in question will be purchased abundantly. Another property of this demand function is the fact that the aggregate demand does not depend on the degree of substitution among the products. Finally, if prices are identical among all firms, the aggregate quantity does not change with the number of products that exist in an industry.

This model presumes that each firm in an industry sells a single good. The profit attained from selling each good is defined as:

$$\pi_i = (p_i - c)(q_i) \tag{3}$$

where c is the marginal cost for producing a unit of the i-th good. For simplicity, this model presumes that there are no fixed costs in the cost function and costs are homogeneous among firms. The firm's expected behavior is to choose a price that maximizes its profit. By substituting equation 2 on equation 3, taking the derivative with respect to price and setting it to zero, the game theoretical price for the i-th good will be:

$$p_i = \frac{nv + \gamma \sum_{j=1, j\neq i}^{n} p_j + c(n + n\gamma - \gamma)}{2(n + n\gamma - \gamma)} \tag{4}$$

When a firm merges to other firms, the merged party turns into a multi-product firm. This new firm sells m products while the remaining firms sell $m - n$ products. The profit of the multi-product firm will be the sum of the attained profits when considering all goods involved in the merge. Game theoretical prices are obtained by taking the derivative of both products and setting it to zero, the results will be:

$$p_I(m) = \frac{c(n\gamma(4n - 2m - 1) + 2n^2 + \gamma^2(2n^2 - nm - 2n - m^2 + 2m)) + nv(2n + \gamma(2n - 1))}{\gamma^2(2n^2 - nm - 2n - m^2 + 2m) + 2\gamma n(3n - m - 1) + 4n^2} \tag{5}$$

$$p_o(m) = \frac{c(n\gamma(4n - m - 2) + 2n^2 + \gamma^2(2n^2 - nm - 2n - m^2 + 2m)) + nv(2n + \gamma(2n - m))}{\gamma^2(2n^2 - nm - 2n - m^2 + 2m) + 2\gamma n(3n - m - 1) + 4n^2} \tag{6}$$

where equation 5 is the price of the multi-product firm and equation 6 is the price of the outside firm (that is, the firm that is not in the merging party). Notice how both profits are in terms of the products sold by the merging party. The higher the number of products sold by the multi-product firm, the higher are profits for both types of firms. This suggests that the consumer surplus decreases in the presence of a merger[4].

[4] To the reader that is unfamiliar with the consumer surplus, it is described as the amount of utility obtained by a consumer after a transaction. A simple formula for it would be $CS = \sum_{i=1}^{n} U(q_i) - \boldsymbol{q} \cdot \boldsymbol{p}$, where $\boldsymbol{q}$ and $\boldsymbol{p}$ are vectors for quantities and prices respectively. With quantities held constant, an increase in prices leads to a decrease in CS.

2.2 Cournot competition with differentiated products

In this paper, we also consider the agentization of the Cournot competition with differentiated products model. The utility of the representative agent is identical to the Bertrand case with the exception that instead of inserting equation 2 into the profit function, the indirect demand is inserted. This indirect demand is given by the following equation:

$$p_i = v - \frac{1}{1+\gamma}(nq_i + \gamma \sum_{j=1}^{n} q_j) \tag{7}$$

In this model, the firm's expected behavior is to choose a quantity that maximizes its profit. The game theoretical quantity for the i-th good will be given by the following equation:

$$q_i = \frac{(v-c)(1+\gamma) - \gamma \sum_{j=1, j\neq i}^{n} q_j}{2(n+\gamma)} \tag{8}$$

Like the Bertrand case, we consider that a firm merges to other firms, which leads to the creation of a multi-product firm that sells m products. The remaining firms sell $(n-m)$ products. The game theoretical quantities that are obtained from this new market arrangement are given by the following equations:

$$q_I(m) = \frac{(v-c)(1+\gamma)(2n+\gamma)}{(2n+2m\gamma)(2n+\gamma(n-m+1)) - mn\gamma^2 + \gamma^2 m^2} \tag{9}$$

$$q_o(m) = \frac{(v-c)(1+\gamma)[(2n+2m\gamma)(2n+\gamma(n-m+1)) - mn\gamma^2 + \gamma^2 m^2 - 2\gamma(2n+\gamma)]}{(2n+\gamma(n-m+1))((2n+2m\gamma)(2n+\gamma(n-m+1)) - mn\gamma^2 + \gamma^2 m^2)} \tag{10}$$

where equation 9 is the quantity sold of each product in the multi-product firm portfolio and equation 10 is the quantity sold by firms that are outside of the merging party. These quantities are conceptually similar to the Bertrand case, because it assumes that the firms are in an symmetric equilibrium. However, mergers under a Cournot competition are not necessarily profitable. This happens because merged firms produce a smaller quantity than non-merged firms, and unless products are extremely differentiated, which is associated to a small γ, the firms that are outside the merger will compensate the smaller production from the multi-product firm in order to increase their own profits.

3 The agent-based model

The last section gave a quick overview regarding the analytical model of mergers. Describing the agentization of the model is the purpose of this section. As is going to be shown through the Overview, Design Concepts, and Details (ODD) protocol based on [12], the agentization of this model is going to occur with respect to firm behavior. Instead of firms that have access to perfect information and rationality, firms are rationally bounded. With this we are effectively relaxing some of the hypotheses of the original model.

Purpose and patterns

The purpose of the model is to reproduce the game theoretical results of a differentiated products Bertrand competition. Mergers lead to market concentration, which leads to a general increase in prices.

Entities, state variables and scales

The entities of the model are firms that are engaged in the competition. In this version of the model, geographic space is not relevant. Simulations are run from 1500 to 4000 periods. Every firm has the following state variables:

- *current price*: from which it draws price bids every period in the Bertrand case;
- *current quantity*: from which it draws quantity bids every period in the Cournot case;
- *cost*: the cost of producing a single good.

Process overview and scheduling

In the Bertrand case, at every period, firms draw price bids which determine quantities. After these quantities are determined, profits are calculated and saved on their memories. When prices are stable, firms engage in the merging process and a new price adjustment phase begins. The Cournot case is quite similar to the Bertrand one, the difference is regarding to the bids that are drawn by firms: instead of drawing price bids, firms draw quantity bids. For the sake of simplicity, we focus on the explanation of the Bertrand case.

Design concepts

Firms try to learn prices that maximize their profits adaptively. Since firms are not able to see information from their competitors, the interaction is only through indirect means because their prices affect demand. From the learning process and adaptation, game theoretical prices emerge.

Initialization

In this version, firms are identical when considering current prices, costs and learning parameters. The only form of heterogeneity in this model is driven by stochasticity, because bids are chosen randomly, and it's not unlikely that bids are distinct from each other. Chosen values for parameters are presented in the following section.

Submodels

Besides the equation that is responsible for determining the demand, another submodel is defined for firm behavior. The learning method is based on [7]. It is an adaptation to a line search method which is useful for finding the optima of functions that have a single variable[5]. Firms draw bids from a uniform distribution:

$$bid \sim U(\text{current price} - \delta, \text{current price} + \delta) \tag{11}$$

after their bids are drawn, quantities are determined by equation 2. The results from their profits are saved into one of two lists: the first one is for when a firm bids higher than its current price; the other list is for when a firm bids lower than its current price. After an "epoch", the name given for a learning phase consisting of 30 periods, the firm compares the mean from both lists. If the list related to high prices has a mean profit higher than the list of low prices, then:

$$new\ current\ price = current\ price + \epsilon \tag{12}$$

else:

$$new\ current\ price = current\ price - \epsilon \tag{13}$$

after the firm is done, the lists are emptied and a new learning phase begins. A simple pseudocode, adapted from [7, p.189] is presented for the sake of clarity on algorithm 1.

Another simple submodel for price stability (or equilibrium) is necessary for the initialization of a merger. Stability is defined in terms of moving averages. A moving average considers the current price of three epochs. Let μ_1 be the average of three epochs; after that, μ_2 is defined as the average of the next three epochs. If the absolute value of the difference between these two averages is lower than a threshold, that is $|\mu_2 - \mu_1| \leq \theta$, then prices are stable, which means firms have found an equilibrium. If that is not the case, $\mu_1 = \mu_2$ and a new value of μ_2 is calculated considering the next three epochs.

Because we are interested in the means of current prices before and after mergers, mergers occur only after prices are stable. When two firms merge, the profit of each product is calculated individually, but the firms that are part of the merging party see their profit as the sum of their individual profits.

3.1 Model implementation and parameters

The model was implemented using Netlogo v6.2 ([14]). Its parameters used in simulations were the ones given in Table 1.

Under such parameters, the game theoretical (optimal price) is: 11.54, and the optimal quantity associated with that price is: 29.49. After a merger happens, the optimal price for the merging party is: 16.87; for the non-merging firms, the price will be 13.86. For the Cournot case, the Nash equilibrium quantity is 21.43, while price will be 35.71.

[5] See [11] for an overview of the method.

Algorithm 1 Probe and Adjust

```
1: Set learning parameters: δ, ϵ, epoch_length
2: counter ← 0
3: returns_up ← [] (An empty list associated with current price raises)
4: returns_down ← [] (An empty list associated with current price decreases)
5: Do forever:
6: counter ← counter + 1
7: price_bid ~ U(current_price − δ, current_price + δ)
8: profit ← Return of price_bid
9: if bid_price ≥ current_price then
10:     Append profit to returns_up
11: else
12:     Append profit to returns_down
13: end if
14: if (counter mod epoch_length = 0) then (This means the learning period is over.)
15:     if mean returns_up ≥ mean returns_down then
16:         current_price ← current_price + ϵ
17:         returns_up ← []
18:         returns_down ← []
19:     else
20:         current_price ← current_price − ϵ
21:     end if
22: end if
23: Back to step 5
```

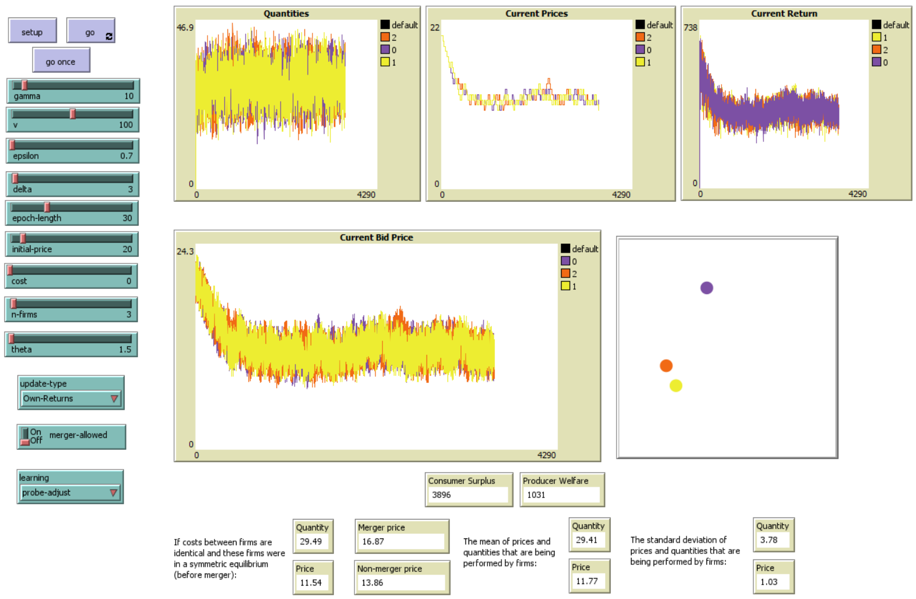
Table 1. Parameters used in simulation

	Value
γ	10
v	100
ϵ	0.7
δ	3
θ	3
Epoch length	30
Initial price	20
Firm costs	0
Number of firms	3

After a merger happens, the quantity of the merging party will be 17.67, while the non-merged party will produce 28.71. Prices of the merged and non-merged parties will be very close, being 3.69 and 3.39 respectively.

In the Netlogo model, there are sliders that determine the value of learning parameters, such as ϵ and δ, demand parameters, such as γ and v, initial prices, firm costs, the number of firms.

Fig. 1. Typical run in the model.



4 Results

In this section we will discuss the results of the model considering the parameters chosen for our simulations. First we will analyze results obtained before mergers and then after mergers. Finally, statistical tests will be run to compare means of current prices between each of the circumstances.

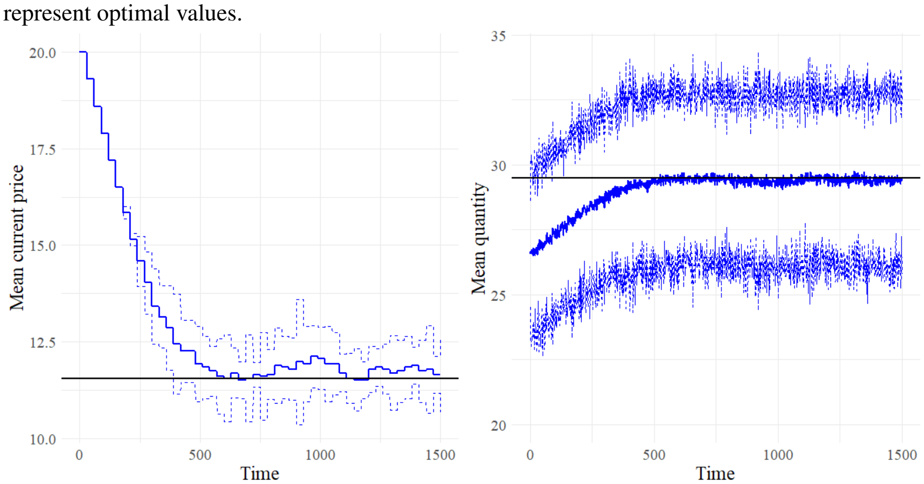
Before mergers

As a first experiment, 10 runs with the given parameters were considered to understand the model's behavior. Every period (step), the mean and standard deviation of prices, quantities and welfare values were taken. The data was grouped by step, so notice that these are the average of means and standard deviations. Figure 2 shows the progression of prices and quantities as time evolves. In our simulations, the stability of prices occur around the 600*th* period.

At the optimum, considering symmetric prices, quantities should be around 29.49. The mean quantity produced by firms follows that very closely, as seen on the right panel of the Figure 2. The mean current quantity is around 29.4 with a standard deviation of around 1.5. Regarding prices, the optimal price is 11.54. Firms in the model have a mean current price of 11.75, higher than the optimal price by a small number; the standard deviation of current prices is around 0.5.

Before mergers actually happen, after a learning period, optimal prices and quantities are achieved by firms. Naturally, the adjustment process depends on the learning

Fig. 2. Mean price (left panel) and mean quantity (right panel) as a function of time. Dashed lines represent the same time series with two standard deviations added. Black horizontal lines represent optimal values.



parameters. For example, if $\epsilon = 2$, adjustment would happen around the $250th$ period. However, the standard deviations of prices and quantities would be higher. Consequently, an adjustment to the the stability parameter (θ) would be necessary to consider the higher standard deviation[6].

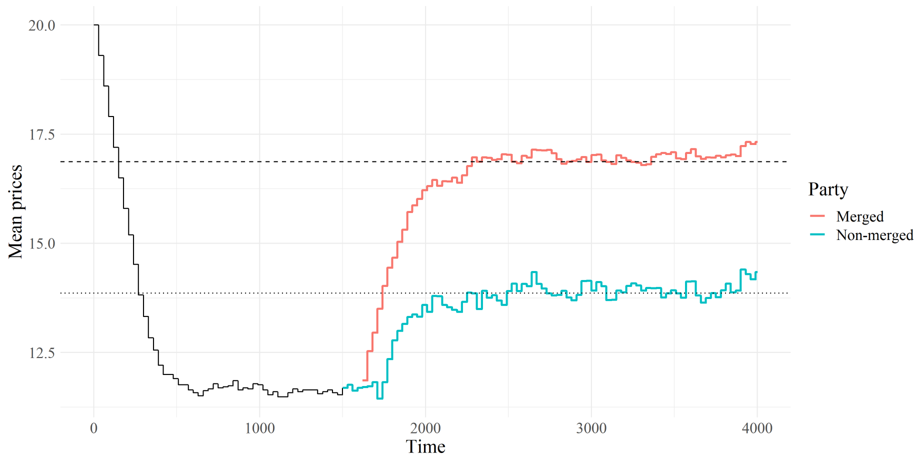
After mergers

The next experiment is related to the effect of mergers on prices. Once again, 10 runs with the given initial parameters were considered, but now mergers happen only starting at the $1500th$ period and the stability condition is achieved[7]. Immediate results are given in Figure 3, which shows prices after a merger has taken place and involves two of the three companies. Increases in prices are present for both groups that are in the industry: the merged party and the non-merged. Following the analytical model, mean prices are around 16.5 with standard deviation of 1.08 for the merging party. In the case of the non-merging firm, mean price is 13.6 and its standard deviation is 0.724.

Because this difference in prices could be due to randomness, statistical tests were conducted to help decide if the difference in means are significant. The result from a one-way ANOVA[8] test suggests that the null hypothesis of equal price means between merged and non-merged parties can be rejected with a confidence level of 95%. When considering a non-parametric test, such as Kruskal-Wallis[9], the result is the same: we reject the null hypothesis that means are equal. This suggests the changes observed after a merger are not only by chance.

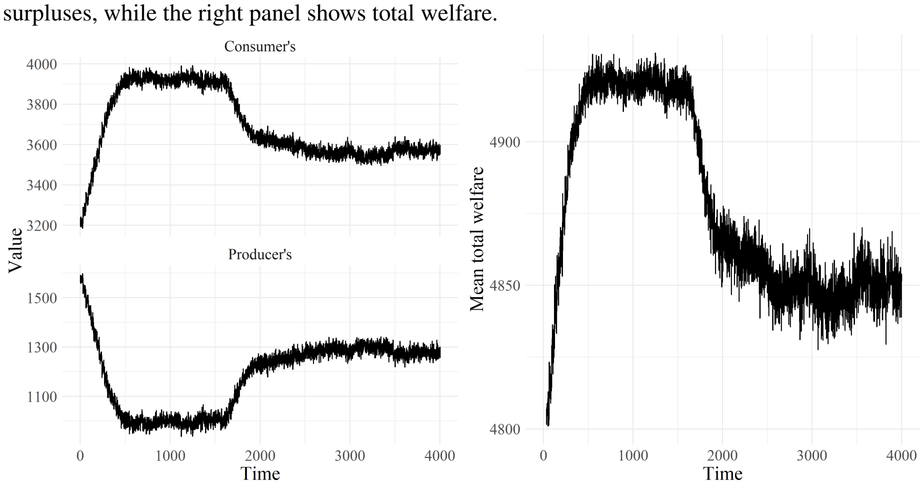
[6] This is relevant only when mergers are allowed.
[7] This means that firms merge around the 1590th period.
[8] See [4].
[9] See [6].

Fig. 3. Mean prices after a merger. Dashed lines represent optimal prices for each party after mergers.



As suggested by the analytical model, a merger leads to an increase in prices for both parties in the market (the merged and the non-merged). If prices are increasing, consumer's and producer's surpluses are both affected. The left panel of Figure 4 shows the evolution of surplus for both parties. Not only consumer surplus decreases after a merger, the total welfare, which is the sum of the surpluses for both parties, decreases as shown in the right panel. This is another expected result in the analytical model.

Fig. 4. Welfare measures time progression. The left panel shows the consumer's and welfare's surpluses, while the right panel shows total welfare.

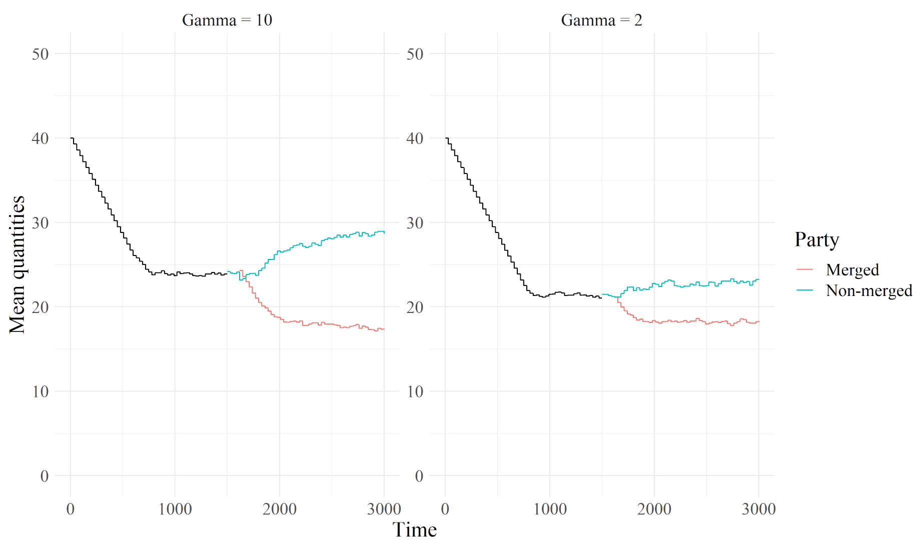


4.1 Results from Cournot competition with differentiated products

This simulation experiment considered an agentized version of the Cournot competion with differentiated products model. In this case, firms use the *Probe and Adjust* algorithm to define their quantities. Firms are initially engaged in competition and try to learn the optimal quantities, that is, quantities that maximize their profits. After the $1590th$ period, firms in the system merge and adjust their quantities considering the new arrangement.

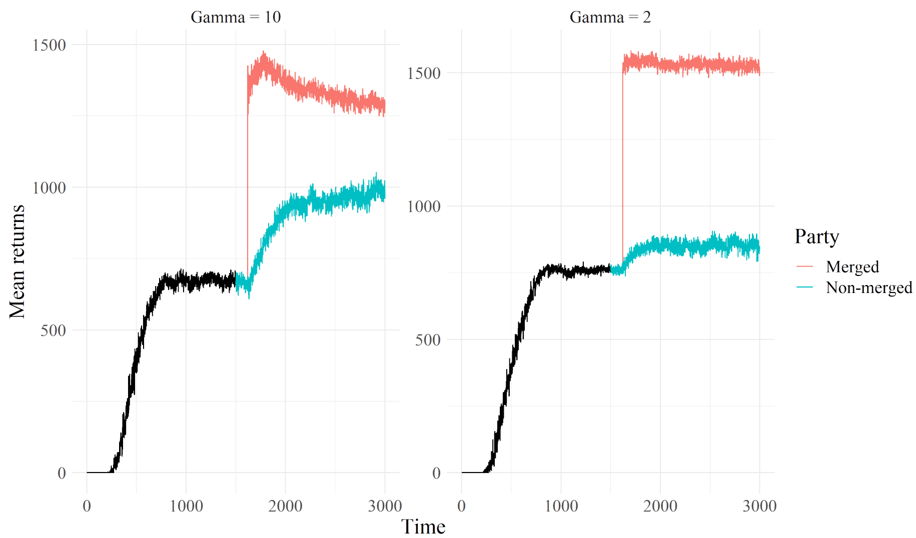
Figure 5 shows the mean quantities chosen by firms in the system as a function of time. Initially, the mean quantities chosen by firms are higher than the Nash equilibrium, considering the chosen parameters. For that reason, quantities are decreasing until they're stable, which happens around the $600th$ period. When firms merge, the multi-product firms decrease their quantities. In response, if $\gamma = 10$, the firm that is outside the merger increases its sold quantity. When $\gamma = 2$, quantities for the merged firm decrease, but the firm that is not part of the merger does not increase its quantity as much. This is the predicted result in the analytical model, which attests the precision of the agent-based version, even when considering that firms are rationally bounded.

Fig. 5. Mean quantities as a function of time



The smaller the value of γ, the higher the prices after a merger. In the case that γ is sufficiently small, the profits of the merging party are increasing. An interesting pattern emerges when $\gamma = 10$: initially, the multi-product firm profit is increasing as time progresses, but it starts to decrease because the firm that is outside of the merger starts increasing its produced quantity. This is happening because the firm that is outside of the merger does not know that increasing quantities will lead to an increase in its profit. Finally, it's noticeable that mergers are unambiguously beneficial to the firm that is outside of the merger in the Cournot case. These results are observed in Figure 6.

Fig. 6. Mean profits as a function of time



5 Concluding remarks and future extensions

Our paper has shown how the agentization of a horizontal merger can be conducted considering firms with imperfect rationality and incomplete information. Even with a rudimentary method, such as the Probe and Adjust, in which firms are basically trying and guessing prices, firms can adapt to their environment and learn optimal prices and quantities in distinct scenarios: without and with mergers.

For future extensions of this work, alternative learning methods should be implemented and compared to the Probe and Adjust. Heterogeneous learning is an interesting extension to this work because different pricing patterns could emerge. For this future implementation, candidate methods could be the least squares method and the gradient learning. Another relevant extension to this work would be the inclusion of spatial competition: an additional parameter could be added to the utility function of the representative consumer in order to denote its preferences according to a firm's distance. Another way to explore spatial competition would be in the ways proposed by [8], in which consumers are uniformly distributed in a unit circle.

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